

A mathematical model for aerodynamic coupling effect analysis of high-rise buildings

Yi Hui^{1,*}, Zhen Liu¹

¹ School of Civil Engineering, Chongqing University, Chongqing, 400030, China

* alihui@cqu.edu.cn

SUMMARY:

High-rise buildings, with similar natural frequencies of sway modes in two orthogonal directions, can have coupled vibrations exhibiting complex fluid-structure interaction and aero-elastic effects, when exposed to strong wind. Two-degree-of-freedom (2DOF) aero-elastic square section high-rise models are tested in wind tunnel to provide data for the analytical modelling, showing clearly the coupling effects between different dofs. This paper studies the coupled vibration with the development of a mathematical model considering both the in-plane and out-of-plane self-excitation force. A new strategy to estimate the aerodynamic parameters and self-excitation forces is proposed. Results showed that the proposed strategy can effectively identify both the buffeting force and self-excitation force, which includes the out-of-plane and in-plane components for each direction. The out-of-plane self-excitation force acting in the along-wind direction induced by the across-wind vibration can become dominating and induce large amplitude vibration in the along-wind direction. The wind induced coupling vibrations for other tall buildings can similarly be studied in detail with the proposed strategy.

Keywords: High-rise buildings, Mathematical model, Coupled vibration

1. INTRODUCTION

The large amplitude wind-induced vibration of high-rise building consequently draws great attentions worldwide. Recently some studies have shown that when high-rise buildings undergo large amplitude vibration, strong coupling effects between the along- and across-wind directions can be observed (Liang et al., 1993; Jauvitis and Williamson, 2004). It was found that the coupled vibrations in two orthogonal directions are closely related to the energy transfer and stiffness ratio of structure (Li et al., 2021; Hao and Yang, 2020). However, the above studies only described the coupling phenomenon between the vibrations in two orthogonal directions. A mathematic approach to describe such behavior of tall buildings has not been developed.

In order to mathematically describe coupled vibrations in across- and along-wind directions, the wind tunnel tests of 2DOF aero-elastic square section high-rise building models are firstly carried out. Then, a mathematical model of the coupling self-excitation force is proposed with the reference to the Scanlan's flutter model (Scanlan, 1978). Parameters of the proposed model will be also identified (Hui et al, 2019).

2. WIND-TUNNEL EXPERIMENTS AND DATA ANALYSIS

Wind tunnel experiment was conducted in a boundary layer wind tunnel at Chongqing

University, China. The testing aero-elastic model is a square prism with height (H) of 0.5 m and width (B) of 0.1 m, representing a 150 m tall building with a length scale of 1:300. The mass ratio of building is 130.61, and Scruton number of the model is 16.41. The natural frequencies in across-wind and along-wind directions are 6.37 Hz and 5.37 Hz, respectively. The system damping ratio is 1.0%.

The incident wind in open terrain was simulated in accordance with code (GB5009-2012) with good agreement to the target wind profile as shown in Figure 1(a). Figure 1(b) shows power spectrum density of along-wind turbulence. Figure 1(c) shows the power spectral density (PSD) of acceleration responses in across-wind and along-wind directions when $U_R = 13.53$. In the across-wind direction, only one dominant peak can be observed suggesting the resonance vibration in the across-wind direction. More interesting results can be checked in the along-wind direction, two distinct peaks can be observed. By examining the frequency, it is found that the first peak corresponds to the natural frequency of along-wind vibration mode. The second peak, however, corresponds to the modal frequency of across-wind vibration mode. It again suggests that the across-wind vibration can significantly affect along-wind responses, indicating the coupling effects between the along- and across-wind vibrations.

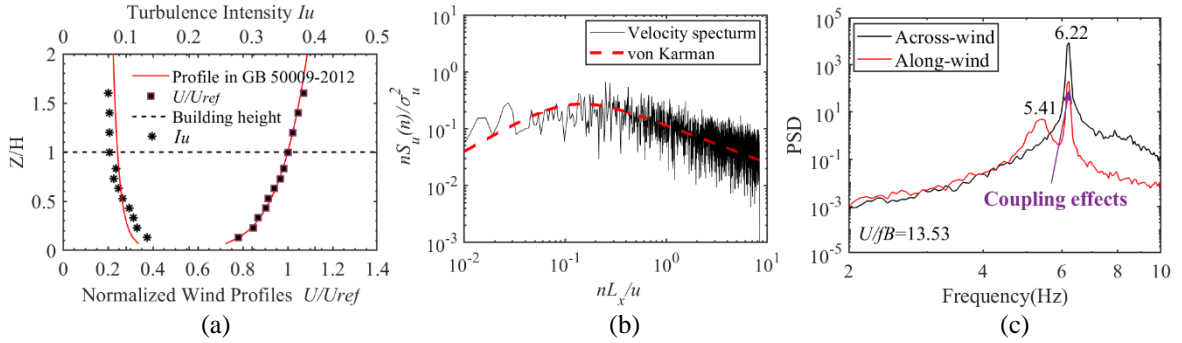


Figure 1. Characteristics of incident wind: (a) Profiles of normalized mean wind velocity and turbulence intensity; (b) Power spectrum density of along-wind turbulence; (c) PSD of acceleration responses at $U_R=13.53$.

3. MATHEMATICAL MODEL OF COUPLING SELF-EXCITATION FORCE

With reference to the Scanlan's flutter model (Scanlan, 1978), the self-excitation force in each direction (along- or across-wind) could be expressed as the displacement and velocity responses in both along- and across-wind directions as follows:

$$M\ddot{x} + C_X\dot{x} + K_Xx = F_X + A_1 \cdot \dot{x} + A_2 \cdot x + A_3 \cdot \dot{y} + A_4 \cdot y, \quad (1a)$$

$$M\ddot{y} + C_Y\dot{y} + K_Yy = F_Y + B_1 \cdot \dot{y} + B_2 \cdot y + B_3 \cdot \dot{x} + B_4 \cdot x, \quad (1b)$$

where x, \dot{x}, \ddot{x} are the along-wind displacement, velocity and acceleration response, respectively. Correspondingly, y, \dot{y}, \ddot{y} represent the across-wind response; M is structure mass. C_X, C_Y, K_X, K_Y are the structure damping and stiffness in two directions; F_X, F_Y are the buffeting force. $A_1, A_2, A_3, A_4, B_1, B_2, B_3, B_4$ are self-excitation force coefficients, which need identification.

$$\text{Let: } F_{XX} = A_1 \cdot \dot{x} + A_2 \cdot x, \quad F_{XY} = A_3 \cdot \dot{y} + A_4 \cdot y, \quad F_{YY} = B_1 \cdot \dot{y} + B_2 \cdot y, \quad F_{YX} = B_3 \cdot \dot{x} + B_4 \cdot x, \\ C_{RX} = C_X - A_1, \quad K_{RX} = K_X - A_2, \quad C_{RY} = C_Y - B_1, \quad K_{RY} = K_Y - B_2, \quad F_{dX} = F_X + F_{YX},$$

where F_{XX} (F_{XY}) represents the self-excitation force acting on the along-wind direction, which is function of the response in the same (orthogonal) direction. Similar definition applies to the self-excitations force in across-wind direction, F_{YY} and F_{YX} . To classify the four self-excitation forces, F_{XX} and F_{YY} are defined as the in-plane self-excitation force. F_{XY} and F_{YX} can be referred as the out-of-plane self-excitation force. Consequently, A_1, A_2, B_1, B_2 are defined as in-plane self-excitation parameters, and A_3, A_4, B_3, B_4 are

defined as out-of-plane self-excitation parameters. While, according to the results shown in Figure 1(c), F_{YX} is negligible, coefficients B_3 , B_4 are then ignored in following study. C_{RX} , C_{RY} and K_{RX} , K_{RY} represent the system damping and stiffness in the along-wind and across-wind direction, respectively. The system parameters of the mathematical model can be obtained through the analysis strategy (Hui et al., 2019).

4. RESULTS DISCUSSION

4.1 Parameter identification in along-wind direction

Figure 2 shows the identified parameter results of along-wind direction at $U_R = 9.79$. Figure 2(a) is the maximum spectral value of identified excitation versus different K_{RX} . According to Ref (Hui et al., 2019), the system stiffness is 941.12 N/m. The structure stiffness is 913.62 N/m. Consequently, the value of A_2 can be obtained as -27.5 N/m according to Eq. (1). Figure 2(b) shows the maximum spectrum curvature of identified excitation versus C_{RX} . The system damping should be around 0.591 Ns/m. Then, the value of A_1 can be identified as -0.063 Ns/m using Eq. (1). The black solid line shown in Figure 2(c) is the PSD of F_{dX} . A prominent peak can be clearly checked, and this peak is related to the across-wind vibration of model, which means the out-of-plane self-excitation force. Coefficients A_3 and A_4 can then be identified by removing this spectrum peak from F_{dX} , using trial and error method (Hui et al., 2019). Finally, A_3 and A_4 are identified as -0.11 Ns/m and -27 N/m, respectively. The dashed line in Figure 2(c) shows the PSD of F_X after removing the out-of-plane self-excitation force.

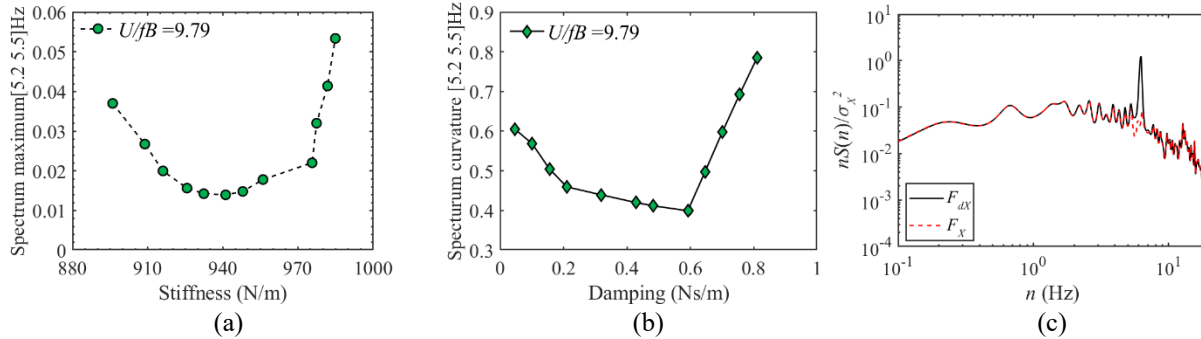


Figure 2. The identified parameter results of along-wind direction: (a) K_{RX} ; (b) C_{RX} ; (c) PSD of F_{dX} and F_X

5.2 Self-excitation force in mathematical model for 2DOF model

Figure 4 shows the RMS of acceleration response with and without in-plane self-excitation forces in along- and across-wind directions. It can be checked that for the along-wind direction, the in-plane self-excitation force plays a role of inhibiting the structure response, which can benefit the structure design. On the other hand, for the across-wind direction, F_{YY} can significantly amplify structure response when $U_R > 9.0$. Such effects coincide with that observed in many previous studies (Kwok and Melbourne, 1981; Song et al., 2019).

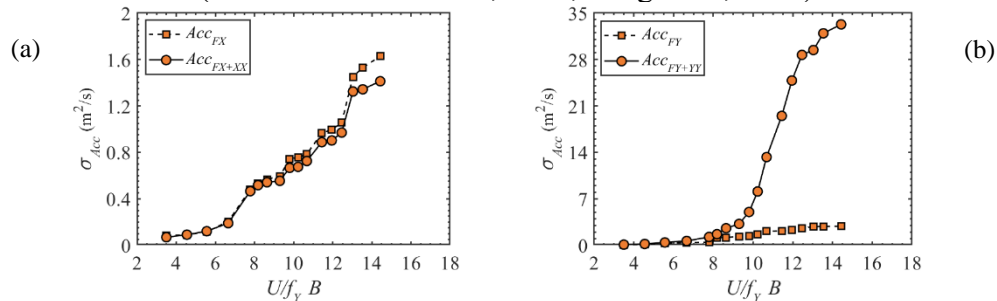


Figure 3. RMS of acceleration response with and without in-plane self-excitation forces

Figure 4 shows the out-of-plane self-excitation force (F_{XY}) in along-wind direction. Figure 4(a) provides the power spectrum of F_{XY} . It can be found that there is a clear peak frequency in spectrum, which coincides with the natural frequency of across-wind direction. Figure 4(b) provides the RMS value of acceleration responses with and without considering F_{XY} in along-wind direction. The RMS value of acceleration responses under $F_{X+XX+XY}$ is about 2.7 times greater than that under F_{X+XX} . It illustrates the out-of-plane self-excitation force has much stronger effect on the along-wind response comparing to that of buffeting force and in-plane self-excitation force acting on the along wind direction.

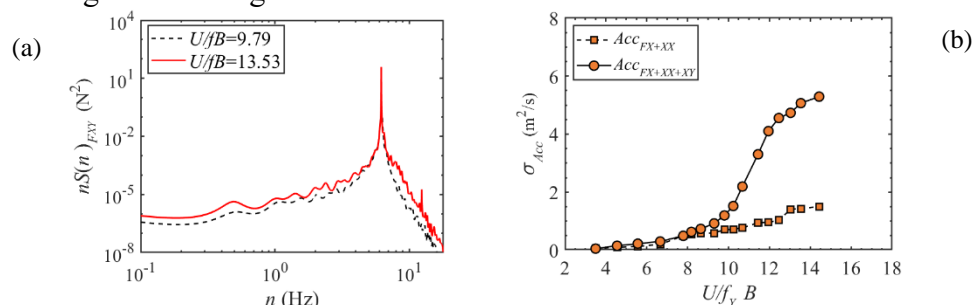


Figure 4. Out-of-plane self-excitation force under $U_R = 9.79, 13.35$: (a) the power spectrum; (b) RMS value of acceleration responses of along wind direction with and without F_{XY} .

7. CONCLUSIONS

In this study, the coupling effect on wind induced vibration of high-rise building is investigated through wind tunnel experiment. The main conclusions are summarized as follows: the in-plane self-excitation force can significantly amplify structure response when reduced wind speed is greater than 9.0 in the across-wind direction. While, in along-wind direction, it plays a role of inhibiting the structure response. The out of plane self-excitation force, however, can significantly amplify on the along-wind response. It means the coupling effect during vibration should be carefully addressed.

ACKNOWLEDGEMENTS

The authors are grateful for the financial supports by Natural Science Foundation of China [grant numbers 52078087, 52221002].

REFERENCES

- GB5009-2012. Load code for the design of building structures. China Architecture and Building Press, Beijing, China; 2012.
- Hao W, Yang QS. Experimental investigation on coupled vibration characteristics of wind-excited tall buildings. Adv Struct Eng 2020; 23(9): 1948-1959.
- Hui Y, Law SS, Liu M, et al. Parameter and aerodynamic force identification of a SDoF System in wind tunnel test. J Eng Mech (ASCE) 2019; 145(1): 04018120.
- Jauvtis N, Williamson CHK. The effects of two degrees of freedom on vortex-induced vibration at low mass and damping. J Fluid Mech; 2004, 509: 23-62.
- Kwok KCS, Melbourne W H. Wind-induced lock-in excitation of tall structures. J Struct Div 1981; 107(1): 57-72.
- Li X, Yu XY, Li QS. Field measurement and validation of structural dynamic parameters of skyscrapers under super typhoon excitation. J Civil Struct Health Monit 2021; 11(3): 609-627.
- Liang SG, Li QS, Li GQ, et al. An evaluation of onset wind velocity for 2-D coupled galloping oscillations of tower buildings. J. Wind Eng Ind Aerodyn; 1993, 50: 329-339.
- Song WW, Liang SG, Song J, et al. Investigation on wind-induced aero-elastic effects of tall buildings by wind tunnel test using a bi-axial forced vibration device. Eng Struct 2019; 195:414-424.

Scanlan RH. The action of flexible bridges under wind, I: Flutter Theory. *J sound Vib* 1978; 60(2): 187-199.